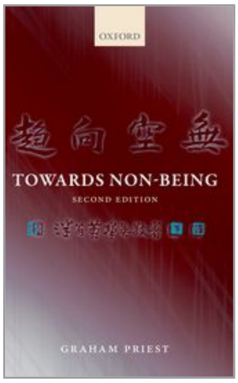

Towards Non-Being
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Mathematical Objects and Worlds

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Abstract and Keywords

Chapter 7 provides a noneist account of mathematical and other abstract objects, and of worlds (possible and impossible). It then discusses a number of objections, such as that this is just a form of platonism in disguise.

Keywords: mathematical objects, noneism, possible worlds, Platonism

7.1 Introduction: Kinds of Non-Existent Object

Purely fictional objects are not the only kind of non-existent object. Arguably, another such class comprises abstract objects, and particularly mathematical objects. Not all noneists have taken these objects to be non-existent. As we noted in Ch. 5, for Meinong himself, and Russell in the *Principles of Mathematics*, these were not non-existent objects, but subsistent ones. However, Routley took abstract objects as non-existent.[1] And the picture of reality whereby it comprises the existent, which are concrete objects in space and time, and, for the rest, the non-existent, has an appealing cleanness about it.[2] In this chapter we will start by looking at the treatment of abstract objects as non-existent.

In previous chapters, much use has been made of the notion of worlds, their properties and relations; indeed, these have been central to the analysis of intentionality offered in the book; and a natural question concerns the status of worlds themselves. Worlds have a vexed status, (p.135) even without noneism. But if one is a noneist, an obvious possibility is that all worlds, with the exception of the actual world, are non-existent objects. This was, in fact, Routley's view. Worlds as non-existent objects will be the subject of the next part of the chapter.

Purely fictional objects, abstract objects, and worlds are three kinds of non-existent objects. The categorization is not meant to be exhaustive[3]—or exclusive, for that matter—as we will have occasion to note. But they are some of the most important kinds; and our discussion of them will raise a number of questions for, and possible objections to, noneism; for example: How does one know about such things? Is noneism really platonism in disguise? In the rest of the chapter, we will look at these issues.

7.2 Abstract Objects

Abstract objects have a notoriously troubled locus in philosophy. Properties, relations, propositions, and, above all, mathematical objects, have an ontological and epistemological status that is highly problematic. All accounts seem to face difficulties.[4] Platonism of some form is, perhaps, the default position. And a noneist can certainly endorse a platonist account. Meinong himself subscribed to a form of this. For him, as we have already seen, abstract objects do not exist, but they do subsist; that is, they have a distinctive form of being. But for a noneist, a simpler view beckons; it certainly summoned Routley. Abstract objects are just another kind of non-existent object. This at least accounts for the fact that there seems to be a very great difference in kind between ordinary concrete objects and abstract objects. The difference between existence and non-existence is about as great as can be! The question is whether a noneist account stands up to closer inspection.

Let us start with the question of what, exactly, an abstract object is. The answer to this is by no means obvious. A natural first suggestion is that abstract objects are ones that do not enter into causal chains with you, me, and the things that we, in turn, interact causally with. Such an account is problematic, however. For example, a modal realist, such (p.136) as Lewis, takes worlds other than the actual to satisfy this criterion. Yet these worlds are not abstract objects, but physical ones, just like the actual world. The noneist has a different problem. Purely fictional objects, like Holmes and Zeus, do not enter into causal chains with respect to us. 'x doing such and such caused y to do such and such' would appear to be an existence-entailing property. Since these objects do not exist, they cannot enter into causal relations; but they are not abstract objects, at least as usually conceived.

For a noneist, a major difference between standard purely fictional objects and abstract objects would seem to be in the *mode* of their existential status. Holmes and Zeus do not exist, but they could have done. There are possible worlds that realize the Holmes stories, and in those Holmes does exist. The status of being an abstract object, by contrast, would appear to be a non-contingent matter. To exist is to be concrete. Are there worlds in which, say, 3 is a concrete object? Yes, as we saw in Ch. 4, there are worlds in which anything can be realized. But the world in which one can hold 3 in one's hand hardly seems to be a possible one. Thus, we may take it that an abstract object not only does not exist, but necessarily does not exist.

This suggests taking the necessity of existential status to be the defining criterion of being an abstract object. This account, however, is also problematic, given what has already been said about purely fictional objects. The box in *Sylvan's Box* has contradictory properties. It is natural to suppose, therefore, that there is no possible world where it exists. Yet it is not an abstract object: it's a box (in the story). One may avoid this particular objection, as I would be inclined to, by simply accepting the fact that there are possible worlds at which contradictions are true. But the point is more general than this. Whatever one takes the correct logic to be, one can construct a story in which there are objects that have logically impossible properties. In 6.4 I noted, for example, the possibility of a purely fictional object, a person, who violates the logical law of distribution. The person satisfies the condition $A(x) \wedge (B(x) \vee C(x))$, but not the condition $(A(x) \wedge B(x)) \vee (A(x) \wedge C(x))$. This is a person (in the story), not an abstract object, but one who exists only at impossible worlds.

Perhaps more success can be had with a counter-factual criterion which, in effect, combines the previous two proposals: an abstract object is one such that, *if it did exist it would still not* (p.137) *causally interact with us*. Conversely, a concrete object is one such that, *if it did exist, it would causally interact with us*.[5] Holmes, were he to have existed, would have entered into causal chains with us. We could have seen him entering and leaving his rooms in Baker St. He is, therefore, a concrete object. Sylvan's box, also, had it existed, would have entered into causal chains with us. Thus, for example, in the story, Nick holds it in his hands. Therefore, it also is not an abstract object. But consider an object that we would naturally take to be abstract, say the number 3. The simplest way to accommodate the claim that this exists is to suppose that the world is such as the traditional platonist takes it to be. If the noneist is right about abstract objects (so that they do not exist) and the existential status of abstract objects is necessary (so that they cannot exist) then such a world is not a possible world. Never mind; worlds of this kind realize the antecedent of the conditional, but it remains the case that 3 does not enter into causal relationships with us at such worlds. Platonists do not normally think that one can see or hold 3. Of course, there will be other, impossible, worlds where we do enter into causal relations with 3. There are worlds, for example, in which 3 is a cat, and so can be stroked. But these are much more bizarre than the usual platonic picture, and so not the worlds relevant to the counter-factual. Thus, if 3 were to exist, it would still not interact with us causally. That is, it is an abstract object.

The account, then, gives us a plausible understanding of what an abstract object is. It should be noted, though, that on this account an abstract object can be purely fictional. Thus, for example, suppose I tell a story about some (actually non-existent) object which is incapable of entering into causal relationships with us. This is a purely fictional object, but also an abstract object. If it were to exist, we would not be able to see it or hold it. Thus, the categories of abstract and purely fictional objects are not disjoint. But this is no problem: one can, after all, tell a story about 3, or about any other non-existent object, just as much as one can tell a story about an existent object, such as Sylvan. Note, also, that, on this account, the division between abstract and concrete objects is not exhaustive. There is no reason to suppose that one or the other of the conditionals: *if x did exist, x would causally interact with us* and *if x did exist, x would not causally interact with us* is true. What would happen if *x* existed might be indeterminate in this regard.

(p.138) 7.3 Worlds

Let us now turn to the status of worlds. One must distinguish immediately between the worlds themselves and the mathematical representation of them. In Part I of the book I gave a semantics for a language with intentional operators and predicates. The semantics is of a kind familiar in contemporary logic, and deploys the apparatus of set-theory. As such, the objects with which it concerns itself are mathematical ones; and the question of their status has been addressed in the previous section.

But worlds themselves are not the same thing: they are what the mathematical machinery represents. In a similar way, we may represent space and time (and the objects in them) by mathematical structures, such as the real line or Euclidean 3-Space. These are mathematical structures; space and time are not (at least, not in the same sense). If we apply the mathematical representation to space and time, we can do so because the two share a common structure. By figuring out the structure of one (the mathematical one), we can therefore learn something about the other. (I will have more to say about this later in the chapter.) In a similar way, the mathematical semantics of Part 1 of the book provides a representation, not of space, but of language, the extra-linguistic, and the relationship between them. And if it is correct, something must be represented by it, and a something that shares the relevant structure. The extra-linguistic wing of the relation includes the worlds themselves, their properties and relations.

What is the status of their being? As far as I can see, the preceding chapters are compatible with any answer one might wish to give to this question. One might, for example, be a realist of Lewis's kind,[6] and take worlds to be concrete objects of the same kind as ours, just not actual. Alternatively, one could take non-actual worlds to be abstract objects of some kind, such as sets of sentences, or constructions out of properties or universals.[7] Of course, if one subscribes to the account of abstract objects of the previous section, this will collapse into a noneist account; but it is quite possible for a noneist to give a different account of abstract objects, as I have noted.

One does not, of course, have to suppose that all worlds have the same status. The actual world is naturally thought of as a special case (though not necessarily, as modal realism reminds us). But one might suppose also, (p.139) for example, that possible worlds and impossible worlds have different sorts of status. Thus, one might be a modal realist about possible worlds, but take impossible worlds to be abstract objects. However, I know of no good arguments for distinguishing between the status of different kinds of non-actual worlds in this way—just as there seems to be no good reason to distinguish between the status of physically possible worlds and that of physically impossible (but logically possible) worlds. A simple uniform policy therefore recommends itself.[8] And in the context of noneism, the obvious policy is to take all worlds other than the actual to be non-existent objects.

This does not necessarily mean that they are abstract objects. Indeed, if one applies the criterion of the last section then, at least for the most part, they are not. The worlds that realize the Holmes stories are replete with things that, were they to exist, would be standard physical objects, like people and hansom cabs. Were these worlds with their denizens to exist, we would be able to interact causally with them. Just as Lewis claimed, then, these worlds are just like the one in which we live—or they would be if they existed. There can, of course, be unusual worlds; for example, worlds where nothing exists. It is not clear that, if this world were realized, it would enter into causal relationships with us. Maybe, then, one should take such a world to be an abstract object. But at any rate, all worlds other than the actual have the uniform status of non-existence.

What of their properties? We have made use of a number of these in previous chapters. We have distinguished, for example, between possible, impossible, and open worlds. The relationship $\Vdash^{\pm}$—that is, the one represented by this relationship in the formal semantics, where the interpretation, $\mathfrak{I}$, in question is the veridical one—is a relationship between worlds and other things. To be specific, since statements concerning it are of the form $w \Vdash^{\pm}_{s} A$, $\Vdash^{\pm}$ is a three place relationship. The first argument is a world, the other two arguments, s and A, are a function and a sentence (type). These are abstract, indeed mathematical, objects. Now, none of the properties and relations at issue here is existence-entailing. To say that a world is possible, for example, does not entail that it exists, any more than to say that an object is possible does. Attributions of modal status are logical attributions, like statements of identity. And $\Vdash^{\pm}$ is not existence-entailing, either; what I have already said about abstract objects (p.140) delivers this fact for at least the last two argument places, and the first is no different. Thus, there is no problem about taking these properties of the worlds in question to be properties they have at the actual world.

7.4 Five Objections

Given the preceding discussion, we can now turn to some objections against noneism, and particularly against a noneist account of abstract objects. There are five natural ones that I will discuss, which are as follows.[9]

1. If some objects do not exist, they cannot enter into causal connections with us. How, then, can we refer to them or speak of them at all?—which a noneist obviously requires us to be able to do.[10]

2. Similarly, since they do not enter into causal connections with us, how can we know anything about them?—which we certainly can if noneism is right.

3. According to the account given, fictional objects and mathematical objects are of a kind: non-existent. Yet, they seem to be quite different sorts of things. For example, we can make up truths about fictional objects as we go along, not mathematical objects. So how can this be?

4. We often apply mathematics to tell us about concrete objects, like shopping, bridges, microchips. How can non-existent objects possibly tell us anything about things that do exist?

5. The noneist and the platonist hold that some objects do not enter into causal relationships with us. They disagree about whether or not they exist, though. But in the end this is just a difference of terminology. When the noneist says that something is an object, the platonist says that it exists; when the noneist says an object exists, the platonist says that it is concrete (and exists). The noneist is just, therefore, a platonist in disguise.

In the rest of this chapter, I will take up each of these points in turn.

(p.141) 7.5 Referring

Non-existent objects do not enter into causal relationships with us, it is true: causation is, as we have noted, an existence-entailing relation. But reference does not require causation. This is obvious in the case of definite descriptions. In these, we single out an object in virtue of its being the unique one satisfying a certain condition; if something is the unique thing satisfying such a condition, we refer to it accordingly. Existence has nothing to do with it. For indefinite descriptions —or even definite descriptions, when no unique thing actually satisfies the defining condition—things are slightly more complex. As we saw in 4.6, the reference is non-deterministic—meaning that factors beyond the semantics fix the reference. Primary amongst these is context, and specifically the intentional acts of the utterer. But none the less, causation need not be required for the appropriate intention, as we will see in a minute.

So much for descriptions. What of proper names? How proper names name is a hard matter. The causal theory of naming has various problems,[11] but let us, for the sake of the present discussion—since the worry is specifically about causation—assume that some version of it is correct. According to this theory, an object is picked out and baptized as *nn* by some agent, *a*. The referent of '*nn*' is picked up by any person, *b*, who talks to *a*, by any person, *c*, who talks to *b*, and so on. Now, causation certainly enters into the transmission of a name's referent; but the causation here is between actual speakers. Supposing that some objects do not exist in no way threatens this. And causation is not required for a baptism—else one could not refer to future objects, which one can. One can, of course, point to a physical object, and so interact with it causally. But one can also pick it out with a description.[12] '*nn*', thereafter, refers rigidly to the object thus selected. Again, non-existent objects in no way threaten this picture. Definite descriptions can be used to pick out non-existent objects just as much as existent ones: 'the object represented by Doyle as living in Baker St, etc., etc.'

Taking intentionality seriously does add an extra dimension to the possibility of baptism, however. As we have seen, picking out an object to name may be performed, not only by a physical act of pointing, but by a mental act of pointing—by simply thinking of the object. Thus, suppose (p.142) that there are two people in front of me. By a simple mental act, I can intend one of them rather than the other. We might call this *primitive intentionality*. In this case, there is, of course, a causal interaction between the intender (me) and the intendee. But causation is playing no essential role here: there is exactly the same kind of causal link between me and the non-intended person. There can also be situations where there is no causation of any kind. Thus, for example, I can close my eyes and imagine a scene with two people in it. By an act of pure intention, I can focus on a particular one of these. The intended objects are still, in this case, spatially discriminable (at least in subjective space). But this does not need to be the case either. Suppose, for example, that you tell me about two Ancient Chinese philosophers, Li and Lu. I can intend either of these at will. I may not know anything about them apart from what you have told me. Indeed, you may have told me exactly the same thing about the two (they were philosophers, lived in the Sung dynasty, in the city of Xian). I can still intend whichever I choose. In this case, I know, at least, that one was Li and the other Lu. But, it seems to me, even this minimal amount of distinguishing information may be absent. An act of pure intention can intend an object when there are other indiscriminable objects. How is this possible? That I think, is the nature of the beast. It must be possible, however, because it has actually been done. As I noted in 4.4, the positive and negative square roots of -1, $+i$ and $-i$, are completely indiscriminable in complex arithmetic. (It would make no difference if what we now call '$+i$', we called '$-i$', and vice versa.) But we can intend $+i$ rather than $-i$. Of course, we now have the names to differentiate the two complex numbers; but it was not always thus. At some stage, some mathematician or committee of mathematicians, must have *chosen* one of these objects arbitrarily and called it '$+i$'. Acts of pure intention, it would seem, can be very powerful.

Since intention is a mental act, one might well worry that it falls foul of Wittgenstein's private language argument (*Philosophical Investigations* §§243 ff.); but it does not. In the situation with which Wittgenstein is concerned, a putative act of reference is brought about by fixing on an essentially private object. There is then no public criterion for making a mistake. In such cases, he argues, no act of reference has been performed. Non-existent objects are not, however, private. They are as public as existent objects. And I can focus my attention on one of them, just as much as I can focus my attention on one of a group of people in front of me. In virtue of what I say to you, you can refer to the same thing. It is then (p.143) perfectly possible for me to make mistakes about which object I originally picked out, which mistakes may be picked up by you. 'Yesterday we were talking about an object you called "Holmes", who lives in Baker St, etc. Now you are telling me that he lives on Olympus, drinks nectar, etc. You've got your wires crossed here: you're talking about Zeus.'

Let me finish this section with a brief discussion of Putnam's 'model-theoretic argument'.[13] Given any theory with a model there are models with different domains that are isomorphic, and so elementarily equivalent, to it. Hence, the theory itself cannot fix what it is about. Putnam uses this as an argument against realism. More recently, Wang (2004) has used it as an argument against noneism. The argument goes as follows. Take any theory, T, the domain of the interpretation of which, according to the noneist, contains some non-existent objects. As long as there is an infinite number of existent things, there will be an isomorphic interpretation of T the domain of which contains only existing things. Hence, noneism is a hypothesis of which we have no need.

I am in agreement with a number of commentators[14] that the fact that a theory has models that are clearly pathological shows that it takes more than the set of sentences to determine its intended interpretation. And it is clear that the model Wang describes is pathological. One way to see this is to note that one of the sentences in T is to the effect that some things do not exist: $\mathsf{S}x\neg Ex$. Hence, to maintain that the correct interpretation of the theory is one in the domain of which all things exist is self-referentially inconsistent. It is not formally inconsistent, of course; for in this interpretation the existence predicate, E, is interpreted so as to apply to only *some* of the existent objects. In other words, it does not have its intended meaning. But to point this out is to give the game away.

This does not, of course, answer the question of what, exactly, it is that makes an interpretation of a theory the correct interpretation. The natural, and, I think, correct answer to this question is that it is the reference relation: the names in the theory must refer to the correct objects.[15] But what determines this? A standard position[16] is to argue that it is some causal connection between the speaker of the language and the (p.144) object in question, perhaps during the process of baptism involving the name, which determines the referent. This particular answer, as Wang points out, is not available to a noneist. But as we have already observed, even with an object that we do perceive, and so causally interact with, there may be more to the matter than this. Given all the objects in my perceptual field, I can focus my mental attention on just one of them. For example, you can be talking to someone (say, at a party), but really wanting to listen to a conversation that is going on behind your back. In such circumstances, you focus your mental attention on that conversation, though it is not 'perceptually dominant'. Exactly the same can be done with vision. And, again as observed, such attention can single out an object for baptism even in a field of objects that is not brought to your attention causally.

Putnam concedes that his argument can be finessed if one is allowed to appeal to the power of primitive intentionality.[17] But he calls this a 'mysterious faculty of the mind' (Lewis dubs it 'noetic rays'[18]) and complains that it should be rejected by any naturalistic (and sensible) philosopher. I do not see why. If there is a naturalistic account of mental functioning (which I presume there is), then there is a naturalistic account of my undoubted ability to focus my mental attention on a part of, or aspect of, what is phenomenologically present to me. This gives us an account of why it is that the model-theoretic argument fails to work.

7.6 Knowing

The second objection flagged was to the effect that, since we cannot enter into causal connections with non-existent objects, we can know nothing about them. A similar objection is, of course, frequently raised against platonism. It seems to me that any reply to the objection given by a platonist could be adopted—with just as much (or as little) success—by a noneist. But the noneists have other strings to their bow as well.

There is no unique way that one comes to know of the properties of non-existent objects. Depending on the object and the properties in question, there are many ways. For a start, I get to know that the (non-existent) man next door is such that I fear him, by introspection. ((p.145) I am not suggesting that this is infallible.) I may get to know that the man next door is feared by you by being told. I get to know that Holmes was characterized by Doyle in certain ways by reading the stories.

Another way in which one comes to know some of the properties of non-existent objects is (as Quine suggested about abstract objects) by hypothesis and confirmation. We formulate a theory about how these objects behave and evaluate it according to the normal canons of theory-evaluation, such as simplicity, coherence, adequacy to the (fallible) data, etc. I will give an example of this later in the chapter, so will not pursue the matter further here.

The most distinctively noneist way of coming to know about the properties of a non-existent object is via characterization. An object (existent or otherwise) has those properties attributed to it by the CP, and those that follow from this. We know those properties precisely because we know the CP and can infer from it. Thus, Sherlock Holmes was characterized in a certain way by Doyle. We know that he had those properties, since they are part of the characterization. Further, we know that Holmes had a friend who was a doctor, not because Doyle tells us this, but because he tells us that Watson was Holmes's friend, and Watson was a doctor; we infer the rest. These properties are not properties of Holmes at this world, of course. As we have seen, a characterized object has its characterizing properties at the worlds that realize the way that things are represented to be. In the case of Holmes, this certainly does not include the actual world. In other cases, though, it may.

Mathematical knowledge may also be obtained by characterization. Suppose that we have a mathematical object, c. c is characterized by some mathematical theory, $\mathsf{T}(c)$. Since our grasp of the CP is to explain our knowledge of the facts about c, then T should, presumably, be something that can *be* grasped. Hence, it is natural to require that the characterization be axiomatic, that is, in effect, that $\mathcal{T}$ be an appropriate set of axioms. Suppose, for example, that $\mathcal{L}$ is the language of arithmetic, formulated in the usual way, with a single constant, 0. Let $\mathcal{T}$ be a set of arithmetic axioms, say the Peano Axioms. Then $\mathcal{T}$ is a set of claims about 0—and various other entities—that characterize its behaviour. Similar comments apply to other mathematical objects and theories.

Do these characterizations obtain at the actual world, or do they, like the Holmes characterization, hold only at other worlds. Nothing so far said forces us to go either way on this issue. However, there would seem (p.146) to be no particular advantage to supposing that they are true at the actual world. We may therefore treat the cases as alike.[19]

At this point, it is natural to object that this cannot explain our grasp of the properties of mathematical objects, since, in the case of arithmetic, set theory, and similar theories, at least, no axiom system is complete, as we know by Gödel's first incompleteness theorem. Incompleteness per se is not a problem, however. If an axiom system for arithmetic is such that it can prove neither $\psi(0)$ nor $\neg\psi(0)$, this may just show that 0 is an incomplete object: 0 simply fails to satisfy both $\psi(x)$ and $\neg\psi(x)$, just as Sherlock Holmes fails to satisfy both *is left-handed* and *is right-handed* (see 6.4).

However, there is also the stronger version of Gödel's theorem, according to which certain sentences are not only not provable in the axiom system, but can be shown to hold none the less. If this is the case, our grasp of the properties of, say, 0, goes beyond any axiomatic characterization. In answer to this, there are two possible replies.

The first, and obvious, one is that our logic is second-order. As is well known, the second-order characterization of arithmetic is categorical, and so the problem does not arise. There is another possible—and much less orthodox—reply, though. This is to point out that Gödel's first incompleteness theorem claims only that *consistent* (first-order) theories of arithmetic are incomplete. But inconsistent noneist objects are quite possible, so to speak, as I have already observed. It is also well known that there are complete inconsistent theories of arithmetic.[20] Moreover, given that mathematics is a humanly learnable activity, there are arguments to the effect that our arithmetic is both axiomatic and inconsistent. Since these arguments may be found elsewhere, I will not pursue them here.[21] What they show, if correct, is that arithmetic is inconsistent, in which case the problem posed by this version of Gödel's theorem lapses.

One might suggest that our knowledge of the properties of, for example, numbers derives not from characterization, but from Quinean hypothesis-and-confirmation. I do not think that this is the case. The question is what the relevant data is against which the theory is to be tested. In the case of pure mathematics, I do not think there is data independent of our characterization. The case is quite different if we (p.147) are testing applied mathematical theories. There we have data about the domain represented. Pure mathematical theories cannot be tested a posteriori. Which brings us to the next objection.

7.7 The a Priori

Objection number three points to the fact that there seems to be a big difference in kind between purely fictional objects and abstract objects, especially mathematical objects. Notably, truths about the former would seem to be a posteriori, whilst truths about the latter would seem to be a priori. What is one to say about this?

The difference in status between mathematical objects and purely fictional objects may be partly explained by the fact that the former are abstract objects and the latter are (normally) concrete. Moreover, as I noted in 7.2, though both are non-existent, the former are necessarily so, whilst the latter are (normally) only contingently so.

But what of the epistemic status of claims concerning the two kinds of objects? For a start, it is not, in fact, the case that all knowledge of abstract objects is a priori. Some is and some is not. For example, we know a priori that the concept *red* (an abstract object) is subsumed by the concept *coloured*. But we do not know a priori that the concept *third planet from the sun* is co-extensional with the concept *planet supporting life*, though this is just as much an (abstract) relation between abstract notions. Nor is it the case that all of our knowledge about fictional objects is a posteriori. It is a priori that Holmes is self-identical.

But concentrate on the sort of examples that people normally have in mind when they make the sort of comparison in question. It would seem that we know a priori that no prime number is the greatest, but not that Holmes lived in Baker St (at least in their respective stories). There certainly appears to be a difference here. But once one looks at the matter more closely, this is not so clear.

The properties of the natural numbers are determined by characterization, say the Peano Axioms. The properties of Holmes are determined, likewise, by characterization—what was written by Doyle. The objects in question have these properties in the worlds realizing the appropriate representations. This is the CP, which is a good candidate for an a priori truth, and is the same in both cases. And it is this that may well be felt to get things wrong. After all, we have to read the Holmes stories to (p.148) know the properties of Holmes (at his worlds); we do not need to read anything to know about the properties of numbers or sets (at theirs). Or, to put it another way, we are—or, anyway, Doyle was—free to make up the properties of Holmes as he went along. We are not free to make up the properties of numbers as we go along, and neither was anybody else.

Arguably, however, the appearance is misleading. In both cases, we may characterize an object purely by fiat. We know a priori that the object so characterized has those properties (at certain worlds), and this is so whether the characterization is provided by what is told in Doyle's novels, or by the Peano Axioms. Doyle made up the characterization of Holmes by fiat. But the Peano characterization also holds by fiat. Presumably, of course, a fiat that took place a long time ago, and only implicitly—in the practice of counting, adding, and so on; but a fiat none the less.[22]

To see more, it is important to distinguish clearly between two sorts of activity. The first is specifying a characterization; the second is figuring out what follows from it. It is the first of these that we normally think of in connection with fiction (making up a story). It can be done entirely ad lib, and it is this fact that gives fiction its feeling of freedom. But, in certain contexts, we evidently do exactly the same in mathematics. For example, Gödel initiated the study of large-cardinal axioms in set theory. Being a platonist, he assumed that some of these axioms are true and some of them are false, independently of our knowledge. But from a noneist point of view, when we postulate a large cardinal axiom, this is just like extending the Holmes stories (see 6.3). And there is no right or wrong way to extend the characterization of sets, any more than there is a right or wrong way to tell a new Holmes story: any way will do (or at least, any way that is compatible with what has gone before).

The second sort of activity, the drawing out of consequences, is what we normally think of first in connection with mathematics. The characterizations of mathematical objects are normally now fixed: mathematics comprises the deduction of what follows from these. There is nothing a posteriori about this: the consequences are governed by the laws of logic. It is this that gives mathematics its a priori feeling. But it is clear that we engage in the second sort of action with respect to fiction as well. When we come out of a film, we argue about the characters, inferring (p.149) from what was shown or said. And the phenomenology of this process is, in fact, very similar to arguing about mathematical objects, though the predicates concerned in arguing about fictional objects are mostly vague, and so interesting cases are rarely cut and dried in the same way that they are in mathematics.

There is at least one further point of dissimilarity that we may observe. Standardly, as I noted in 6.2, not all the representation in a work of fiction is explicit. Thus, though Doyle never tells us (or could have told us) this, it is part of the representation in the Holmes stories that there are no aeroplanes. This is a clear-cut case. But there will be cases that are not so clear. In science-fiction films and stories, for example, it is not always clear what the author intends us to carry over about the laws of nature from real life into the fiction. When we argue about works of fiction, therefore, part of what we may be arguing about is what, exactly, the representation is. This is not the case in mathematics—at least, modern mathematics—where nothing beyond the axioms may be appealed to.

Let me summarize what has been learned in the preceding discussion. There are some important differences between paradigm fictional and mathematical objects, especially concerning the modal status of their existence. There may also be some differences when it comes to a priori and a posteriori knowledge about them; but not substantial differences of the kind one might have thought.

7.8 Applying Mathematics

Let us turn to the fourth objection. How can non-existent objects tell us anything about existent ones? Routley (2003) gestured at a noneist solution to this problem. Facts about non-existent objects can inform us about existent objects since the facts about actual objects may *approximate* those about non-existent objects. Think, for example, of a frictionless plane, an ideal, but non-existent, object. A real plane is not frictionless, but it can be approximately frictionless. Hence, with suitable provisos, if A is true of the ideal plane, A is approximately true of the real plane. Thus, if A is a claim to the effect that an object slides a certain distance across the ideal plane in time t, we can infer that an object will slide the same distance across the real plane in a time $t \pm \varepsilon$, where ε is a contextually determinable real number.

Even if something like this is right, the answer can be only a partial one. For on many occasions we use numbers, non-existent objects, to tell (p.150) us *exactly* how an existent object will behave. Thus, for example, suppose there is a particular particle, say an electron. Suppose that it is moving with a constant velocity $\mathbf{v}$, and that it moves for a time $\mathbf{t}$, through a distance $\mathbf{d}$. Here, $\mathbf{v}$, $\mathbf{t}$, and $\mathbf{d}$ are particular physical, not mathematical, quantities. But each of them can be assigned a certain numerical magnitude, v, t, and d, respectively, by some measuring procedure (using clocks, rulers, etc.). Thus, for example, there is some family of observable properties, P_n, of the distance such that:

(*)

$P_n\mathbf{d}$ iff $d = n$

This establishes a correlation of a certain kind between $\mathbf{d}$ and d. Call biconditionals of this kind *bridge laws*. Now, a law of motion tells us that:

$d = v \times t$

Thus, if we establish by observation, via the bridge laws, that $v = 3$ and $t = 6$, we infer that $d = 3 \times 6 = 18$, and so that $P_{18}\mathbf{d}$. We have used pure mathematical facts to infer something about a physical quantity. Nor are we dealing with ideal objects here; the particle in question is a real-life particle.

How, then, is one to explain the fact that properties of non-existent objects can tell us something about existent objects? Actually, exactly the same question can be posed for platonism, and the answer in both cases is the same. The physical quantities in question have certain properties, and the mathematical quantities have other properties. But we can move between the one and the other because these properties have the same structure, and, specifically, because the correlation established by the bridge laws is an isomorphism. Since mathematical objects may not have their relevant properties at the actual world, we have to understand the bridge laws in a particular way. Thus (*), for example, has to be understood as:

(**)

$P_n\mathbf{d}$ iff $w \Vdash^{+} d = n$

where w is any world that realizes the truths of arithmetic. But the bridge laws still fulfil the function of allowing us to move back and forth between the properties of the physical quantities and those of numbers.

This sort of explanation is quite general. A science, or a branch of it, concerns certain physical quantities, $\mathbf{q}_1, \cdots, \mathbf{q}_m$. These have associated numerical magnitudes, $q_1, \cdots, q_m$, determined by bridge principles of the (p.151) kind (**).[23] In virtue of certain physical states of affairs and the bridge principles, we have some mathematical relation, $F(q_1, \ldots, q_m)$—typically in physics, this would be a differential equation—and working with this we can establish various facts about the q_i, and hence, via the bridge principles, certain physical states of affairs.[24]

Thus, we can use facts about mathematical objects to infer facts about physical states precisely because the two have the same structure. That a certain relation obtains between the mathematical objects can be determined a priori from their characterizations; but which physical relations are isomorphic to which mathematical relations is an a posteriori fact. Its discovery is that of a law of nature. This explanation, which depends simply on there being certain correlations between properties of physical magnitudes and properties of mathematical magnitudes, in no way depends on the numerical magnitudes being existent. All it depends upon is their having the right *Sosein* at the appropriate worlds.

Before we leave the question of applying mathematics, let us return to worlds and their properties. The semantics of Part 1 of the book is a mathematical structure, couched within set theory. But it can be applied to tell us something about worlds, which are not. It is applied in exactly the way that I have just described, though what the mathematical theory of worlds represents is no longer physical reality, but certain non-existent objects. If the relevant aspects of the mathematical semantics are isomorphic to those of worlds, that is, if the representation gets things right, then we may infer facts about worlds from those about their set-theoretic representations via appropriate bridge principles. How do we know whether the set theoretic representation gets things right? In the same way that we test any applied mathematical theory. There are certainly other possible semantics for an intentional language. We determine which it is the most rational to accept in terms of the usual criteria of theory acceptance, such as simplicity, adequacy to the data, and so on. What counts as the data in this case? The sorts of claims that we are (p.152) inclined to make using intentional notions and the inferences that we are inclined to draw concerning them. Such data is, as always, fallible, and may be rejected in the light of overall coherence. None the less, it would be irrational to accept an account of intentionality that ruled out most or much of what we take to be the case concerning intentionality—on that very count.

But why, it might be asked, do we not simply characterize the worlds in question in the appropriate fashion, and infer their properties from characterization? The answer is as follows. One can, indeed, characterize worlds and their properties in any way that one wishes. The worlds, so characterized, have their properties at the appropriate worlds, but these may not be actual. In other words, the statements attributing those properties may not actually be true. But when it comes to semantics, we are after, not just a story, but the truth.

7.9 Platonism

Let us move, finally, to the last objection. This is to the effect that noneism is just platonism in disguise. According to this objection, the translation manual at Table 7.1 shows that a noneist is simply a platonist with an unusual vocabulary.[25]

The objection might well be reinforced by the fact that, in answer to some of the previous objections, the noneist and the platonist can say much the same thing. Note, of course, that for the reduction to be a general one, it must be made with respect, not just to abstract objects, but to all non-existent objects—purely fictional objects, worlds, and so on.

There are many things to be said about this objection. The first is that translation manuals are symmetric. Hence, to suppose that the manual establishes that noneism is reducible to platonism is quite question-begging—at least without further argument. We might just as well say that platonism reduces to noneism. Without such considerations, (p.153) we can just as well say that Plato was a noneist as that Routley was a platonist.[26]

Noneist	Platonist
is an object	exists
exists	is a concrete object

Next, there are, in any case, differences between the two positions. Crucially, the noneist subscribes to the CP; the platonist, at least as usually understood, does not. The noneist claims that any instance of the CP characterizes a perfectly good (though maybe non-existent) object. The platonist does not normally say that an arbitrary characterization characterizes an existent object. Numbers, sets, geometrical lines and points, all these exist. But there is no reason to suppose that any old axiom system—or story—specifies existent objects.

There is a version of platonism that does claim this, however: *plenitudinous platonism*.[27] The plenitudinous platonist holds exactly that there is nothing privileged about the axiom systems for numbers, geometric objects, etc. Every axiom system characterizes equally good abstract objects. The thought that every (consistent) axiom system has a model gives some credence to plenitudinous platonism. The fact that a sentence has a model does not show that it is really satisfiable by certain objects. For example, one can construct a model of the first-order existence '$\exists x$(x is married $\wedge$ x is a bachelor)', though there is no existent object, x, such that x is married $\wedge$ x is a bachelor. Still, models are very much like realities, and the fact that every (consistent) characterization has a model at least gives us a model (so to speak), of what it would be like for every characterization to characterize existent objects (from a platonist point of view).

The confluence between noneism and plenitudinous platonism is still not right, though. A thoroughgoing noneist holds that every characterization characterizes an object. And here, 'every' means *every*. (p.154) Even inconsistent characterizations do this. This diet is probably too rich for even a plenitudinous platonist. Platonists are characteristically very much attached to consistency. So this is an important difference between the noneist and the plenitudinous platonist. Of course, there is still another position out there. This belongs to what we might call the *paraconsistent plenitudinous platonist*. This is a platonist who has foresworn classical logic, and is prepared to endorse a paraconsistent logic.[28] Such a platonist can hold, quite generally, that every characterization characterizes an existent object.

Of course, this sort of platonist cannot hold that every object characterized has its characterizing properties at this world. As we saw in 4.2, if the CP is true at this world, the world is trivial. Hence, the paraconsistent plenitudinous platonist must hold that many of the objects characterized by the CP have their characterizing properties at other worlds. They exist, however, at all worlds.[29]

At this point, the differences between noneism and platonism are disappearing fast. And it must be said that it is the platonist who is making all the concessions. This is a reason to say that the sort of platonism that is left is really noneism in disguise, and not vice versa.

But, since the matter of different possible worlds has now arisen, there is, in any case, still a point where the translation manual breaks down—in modal contexts, and specifically in claims concerning modal status. Thus, consider the claim that Holmes does not exist, but could have done so. This is a claim to which the noneist will assent. The translation is that Holmes is not a concrete object, but could have been. This hardly seems to be true. If Holmes is not a concrete object, what is he? He is not a set, number, property, or other sort of abstract object. And if he is, since abstract objects have their modal status necessarily, it is not possible for him to be a concrete object.

Conversely, Routley did exist, but might not have done so (had his parents not met, for example). The translation of this is that Routley was (p.155) a concrete object, but might not have been. In another possible world, he was a set? Of course, there are worlds where Routley is a set—merely consider the characterizing condition: $x = x$ and Routley is a set. This, like all characterizing conditions, is realized at some worlds; but they are not possible: concrete objects cannot be abstract objects.

A similar point can be made in terms of the explicit characterizations of abstract and concrete objects of 7.2. The number 3 is an abstract object. On the understanding of 7.2, this means that *if* 3 *did exist, it would not causally interact with us*. Under the translation manual, then means: *if* 3 *were concrete, it would not causally interact with us*. But this is false: had 3 been a concrete object, then we would have been able to interact causally with it. Of course, a platonist might try to fashion some other criterion for being an abstract object; but we have seen that such tend to be problematic. And in any case, it remains the fact that the truth of this counter-factual is still not preserved under translation.

There may well be other statements whose truth-value is not preserved under translation. But we have seen enough. Even the attenuated form of platonism, paraconsistent plenitudinous platonism, is still distinct from noneism.

7.10 Conclusion

In this chapter, we have looked at mathematical objects and worlds as non-existent objects. Leaning on what was said in previous chapters, and particularly the account of characterization, we have seen how a natural account of these objects, their properties, and our abilities to refer to and know about them, can be given—an account which is not subject to some natural objections. In fact, in the last three chapters, we have taken in most of the standard objections to noneism. There is one further objection, however. It is not a standard one, but—for my money, at least—it is the hardest. This is the subject of the next, and last, chapter of this part.

Notes:

([1]) Routley (2003). The first part of this paper is to the effect that mathematics is not only noneist, but non-extensional. I shall have little to say about this matter here. I note, however, that if a noneist account of mathematical objects is correct, then the treatment of the CP given in Ch. 4 reinforces

Routley's position on the matter. For if mathematics is about non-existent objects, and if the behaviour of these cannot be understood without talking about worlds other than the actual, then other-worldliness is built into mathematics. But at least as understood in standard modern logical semantics, it is precisely the essential employment of such worlds that is the defining moment of intensionality (with an 's').

([2]) As I indicated in the Preface to the book, this was not quite Routley's picture. For him, only concrete objects presently existing exist. Thus, Socrates and the end of the Earth do not exist. I will not follow him down this path.

([3]) e.g. ideal objects in science, such as frictionless planes and perfect gases, form another plausible class.

([4]) For a discussion of some of the problems, see Priest (1987), 10.4.

([5]) For counter-factuals, see 6.3.

([6]) See e.g. Lewis (1986).

([7]) See e.g. Priest (2001), 2.5–8.

([8]) For further arguments against drawing an ontological distinction between possible and impossible worlds, see Yagisawa (1988).

([9]) Some of these, and some other objections to a noneist account of mathematical objects, are taken up in Routley (2003).

([10]) A version of this objection can be found in Walton (1990), 10.1. In the same section, Walton perpetuates the confusion to the effect that Meinong took non-existent objects to have being.

([11]) For a general discussion of the theory, see Devitt and Sterelny (1987), ch. 4.

([12]) Kripke (1972), 302.

([13]) Putnam (1980); page references are to the reprint.

([14]) e.g. Devitt (1983), Lewis (1984) (page references are to the reprint).

([15]) Putnam (1980), 18, replies to this objection that invoking reference is 'just more theory', and so may itself be reinterpreted. Lewis (1984), 61 f., is right to point out that this is beside the point: the constraint is one that needs to be satisfied, not interpreted as true.

([16]) e.g. Devitt (1983).

([17]) (1980), 4. He calls this, unfortunately in the present context, platonism.

([18]) (1984), 72.

([19]) Note, then, that this makes many claims about both fictional objects and mathematical objects contingent: true at some possible worlds, but not at the actual world—not all though: for example, true identity statements about either kind of object are necessarily true.

([20]) See Priest (1997c) and (2000b).

([21]) See Priest (1987), ch. 3.

([22]) In some branches of mathematics one gets to know the characterization explicitly. For example, one is normally given the axioms of group theory in the first lecture on the topic. But with numbers it is not (normally) like this. One absorbs the Peano Axioms implicitly when one learns to count, add, etc.

([23]) The properties, P_n, employed need not all be observable. Some may be establishable only by inference.

([24]) If one is not a realist about space and time (which I am), one may suppose that there are no actual quantities of space and time, but that talking of such is just a way of talking about certain relationships between objects in space and time. One might therefore object to the particular example I used above. If one does, however, a general account of the same form can still be given. The physical quantities in question are just different (depending on how, exactly, talk of space and time is cashed out).

([25]) An objection to the effect that Routley's view collapses under this translation is made in Lewis (1990). Burgess and Rosen (1997), 224, dismiss Routley's view summarily with an appeal to Lewis's paper. To the extent that they have reasons of their own (p. 188 f.), they perpetuate the confusion that noneists appeal to some kind of being other than existence (see 5.2 and 5.3). They then say that it does not help to replace 'there is' with 'for some': it is not easy, they claim, to understand what the difference is between 'exists' and 'some'. They could simply have reflected on the sentence 'I thought of something I would like to give you as a Christmas present, but I couldn't get it for you because it doesn't exist.'

([26]) Actually, this is one place where the translation manual is certainly not adequate; for Plato held that the forms were not only existent (real), but that they were more existent (real) than concrete objects. No noneist has ever claimed that abstract objects are more existent objects than concrete ones.

([27]) I take the name from Field (1998). The view is advocated by Balaguer (1995), where it is called 'full blooded platonism'. Balaguer defends this platonism against the epistemological objection of 7.6 on grounds very similar to those employing characterization that I used there.

([28]) This position is mooted in Beall (1999).

([29]) A difference between standard platonism and the noneism of the kind explained in this chapter is that, typically, platonists tend to say that the familiar claims about mathematical objects are actually true; this is not the case for noneism of the kind explained. The difference is superficial, however. A noneist could, without too much change to what I have said, hold that standard mathematical objects have their characterizing properties at the actual world; and conversely, as we have just seen, a platonist could hold that mathematical objects have their familiar properties at worlds other than the actual.